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## DISCUSSION ON PROBLEMS IN BUCKLING ANALYSIS OF A CONTINUA

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**Abstract.** In linear buckling analysis the eigenvalue problem, that constitutes the background for estimating critical buckling load, is

$$([S_0] + \lambda[S_\sigma])\{\Delta\} = \{0\} \quad (1)$$

where  $[S_0]$  is the initial global stiffness matrix and  $[S_\sigma]$  is the stress stiffness matrix that is part of the tangential stiffness matrix, both obtained based on linear elasticity. The matrix  $[S_\sigma]$  is obtained by a reference load vector  $\{\bar{A}\}$  and a factor on  $\{\bar{A}\}$  implies the same factor on  $[S_\sigma]$ . The estimated critical buckling load vector is  $\{A\}_C = \lambda_1\{\bar{A}\}$  where  $\lambda_1$  is the lowest eigenvalue for the eigenvalue problem (1). From the assumption of linearity between  $\{\bar{A}\}$  and  $[S_\sigma]$  follows directly, that the critical buckling load vector  $\{A\}_C$  is independent of the size(norm) of  $\{\bar{A}\}$ . This implies uncertainty in linear buckling analysis, and this is illustrated by applying geometrical non-linear displacement analysis, that shows that buckling load also depends on the norm of  $\{\bar{A}\}$ . The relations between the individual stress components in a finite element are unchanged for linear buckling analysis. However, with geometrical non-linear displacement analysis this is not the case, even assuming material linear elasticity. This also give doubts to the estimated buckling load, obtained by linear buckling analysis. The geometrical non-linear buckling analysis is based on the full tangential stiffness matrix  $[\bar{S}_t]$  that is separated in a gamma stiffness matrix  $[\bar{S}_\gamma]$  and a stress stiffness matrix  $[\bar{S}_\sigma]$ . These matrices depend on a reference load  $\{\bar{A}\}$  and therefore the stiffness matrices contain a bar notation and must be determined by iteration. The eigenvalue problem for non-linear buckling analysis is interpreted as an extrapolation along the tangential stiffness matrix

$$([\bar{S}_\gamma] + \lambda[\bar{S}_\sigma])\{\Delta\} = \{0\} \quad (2)$$

Applying this approach for estimating non-linear buckling analysis, comparison to (1) is used to show errors from linear buckling analysis, i.e., for initial uniform and unchanged design the buckling load as a function of the size(norm) of reference load  $\{\bar{A}\}$  is shown not to be constant. A cantilever beam-column and a frame of two beam-columns are used as examples.